Incremental searches & determining initial guess.

Polynomials

use nested formulation

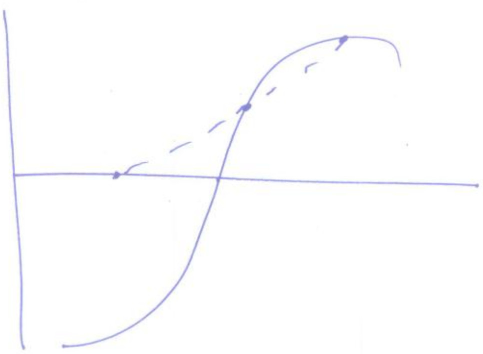
$$f_3(x) = a_3x^3 + a_2x^2 + a_1x + a_0$$
$$= \left[(a_3x+a_2)x+a_1 \right] x + a_0$$

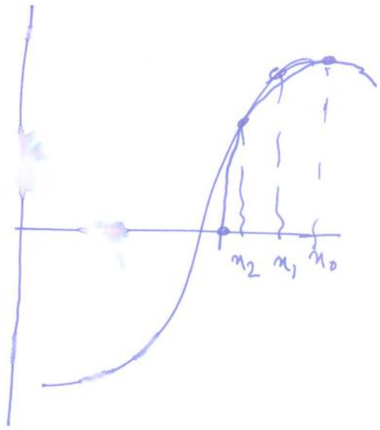
complex roots.

Muller Method

Secant method → uses 2 points, fits a straight line

Muller method → uses 3 points, fits a parabola





eq^n of a Parabola

$$f(x) = a(x-x_2)^2 + b(x-x_2) + c$$

$$f(x_0) = a(x_0-x_2)^2 + b(x_0-x_2) + c$$

$$f(x_1) = a(x_1-x_2)^2 + b(x_1-x_2) + c$$

$$f(x_2) = c$$

convergence
consistency
stability

$$f(x_0) - f(x_2) = a(x_0 - x_2)^2 + b(x_0 - x_2)$$

$$f(x_1) - f(x_2) = a(x_1 - x_2)^2 + b(x_1 - x_2)$$

use $h_0 = x_1 - x_0$

$$h_1 = x_2 - x_1$$

$$\delta_0 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$\delta_1 = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$(h_0 + h_1)b - (h_0 + h_1)^2 a = h_0 \delta_0 + h_1 \delta_1$$

$$h_1 b - h_1^2 a = h_1 \delta_1$$

$$\boxed{\begin{aligned} a &= \frac{\delta_1 - \delta_0}{h_1 + h_0} \\ b &= a h_1 + \delta_1 \\ c &= f(x_2) \end{aligned}}$$

$$\boxed{\begin{aligned} &a x^2 + b x + c = 0 \\ &x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &\text{or} \\ &x = \frac{-2c}{b \pm \sqrt{b^2 - 4ac}} \end{aligned}}$$

$$x_3 - x_2 = \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}$$

$$x_3 = x_2 + \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}$$

$\cancel{x - x_2 \text{ or}}$

$$\epsilon_a = \left| \frac{x_3 - x_2}{x_3} \right| * 100 \%$$

use that sign which matches with sign of b. (7)
for real roots (largest denominator)

Chose two of the three original points nearest the new root estimates.

(2) for both real & complex roots
sequential approach.

$x_0, x_1, x_2 \rightarrow x_1, x_2, x_3 \longrightarrow x_2, x_3, x_4$

Bairstow's method

$f(x) = (x+1)(x-4)(x-5)(x+3)(x-2)$ 5th order Poly.

if we devide by, $x+6$,
quotient = 4th order polynomial
remainder

Algorithm
- guess a value for the root $x = t$
- divide the polynomial by $x - t$
- determine whether there is a remainder or not

- if remainder $= 0$, root is $x = t$
- if remainder $\neq 0$ improve guess
- repeat the procedure till remainder $= 0$

Once one root was found
use the quotient to find other roots.

Bairstow's Method

$f_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots\cdots + a_n x^n \qquad \div (x-t)$

$f_{n-1}(x) = b_1 + b_2 x + b_3 x^2 + \cdots\cdots + b_n x^{n-1}$

$R = b_0$

$b_n = a_n$

$b_i = a_i + b_{i+1} t \qquad \{ i = n-1 \quad \text{to} \quad 100 \}$

$\rightarrow \quad b_0 = 0 \quad$ if $\quad x = t \quad$ is a root.

for complex roots,

divide by $\underline{x^2 - rx - s}$

$f_{n-2}(x) = b_2 + b_3 x + \cdots\cdots + b_{n-1} x^{n-3} + b_n x^{n-2}$

$R = b_1(x-r) + b_0$

$b_n = a_n$

$b_{n-1} = a_{n-1} + r b_n$

$b_i = a_i + r b_{i+1} + s b_{i+2} \qquad \{ \text{for } i = n-2 \quad \text{to} \quad 0 \}$

$R = 0 \Rightarrow b_0 = 0, \quad b_1 = 0$

$b_1(r+\Delta r, s+\Delta s) = b_1 + \frac{\partial b_1}{\partial r}\Delta r + \frac{\partial b_1}{\partial s}\Delta s$

$b_0(r+\Delta r, s+\Delta s) = b_0 + \frac{\partial b_0}{\partial r}\Delta r + \frac{\partial b_0}{\partial s}\Delta s$

$-b_1 = \frac{\partial b_1}{\partial r}\Delta r + \frac{\partial b_1}{\partial s}\Delta s$

A3

let's say

$$\frac{\partial b_0}{\partial r} = c_1$$

$$\frac{\partial b_0}{\partial s} = \frac{\partial b_1}{\partial r} = c_2$$

$$\frac{\partial b_1}{\partial s} = c_3$$

$\therefore$ the partial derivatives can be substituted

solve for Δr & Δs

$$-b_1 = c_2 \Delta r + c_3 \Delta s$$

$$-b_0 = c_1 \Delta r + c_2 \Delta s$$

The partial derivatives can be obtained by a synthetic division of b's.

$$c_n = b_n$$

$$c_{n-1} = b_{n-1} + r c_n$$

$$c_i = b_i + r c_{i+1} + s c_{i+2} \quad \{i = n-2 \text{ to } \phi\}$$

$$\epsilon_{a,r} = \left|\frac{\Delta r}{r}\right| 100\%$$

$$\epsilon_{a,s} = \left|\frac{\Delta s}{s}\right| 100\%$$

$$x = \frac{r \pm \sqrt{r^2 + 4s}}{2}$$

- if quotient is 3^{rd} or higher apply Bairstow's method
- if quotient is quadratic, find directly
- if " is 1^{st} order, $x = -\frac{s}{r}$

Algorithm

1. Polynomial is given, find a_0, a_1, etc.
2. guess r and s
3. find b's. $fn(r, s)$
4. find c's $gn(r, s)$
5. find Δr, Δs
6. New values for r & s
7. stop when $\epsilon_a \leq \epsilon_s$
8. Analyse ~~the~~ ~~remai~~ r, s to find roots
9. Analyse remainder to find other roots.

10 —

1. Direct, analytical
2. Graphical
3. Bisection
4. False position
5. Modified false position
6. Fixed point iteration
7. Newton Raphson
8. secant
9. Modified secant
10. Muller — Polynomials
11. Bairstow — Polynomials

Root location with software Packages

Excell

Goal Seek

set cell — B10
to value — 0
by cchanging cell — A10

OK

Goal Seek Status
Goal Seeking with cell B10
found a solution.
Target value 0
Current value .000637

Solver

MatLab

fzero → root of single function
roots → find Polynomial roots
poly → construct polynomials with specific args
Polyval — Evaluate Polynomial
Polyvalm — " with matrix arg
~~residue~~ —
polyder — Derivative of Polynomial
conv — Multiply polynomials
deconv — Devide Polynomials

= f(x)

Ex.

$f(x) = x^{10} - 1$

$x_l = 0, \quad x_u = 2$

```
>> x0 = [0 2];

>> x = fzero(@(x) x^10-1, x0)

   x = 1
```

$f_5(x) = x^5 - 3.5x^4 + 2.75x^3 + 2.125x^2 - 3.875x + 1.25$

```
>> a = [1 -3.5 2.75 2.125 -3.875 1.25];

>> polyval (a, 1)

   ans =
        -0.2500

>> polyder (a)
   ans = 5.000   -14.0000   8.25  ...

>> b = [1 0.5 -0.5]

>> [d, e] = deconv (a, b)
```

$d \to Q, \quad e \to R$

```
>> roots (d)
```